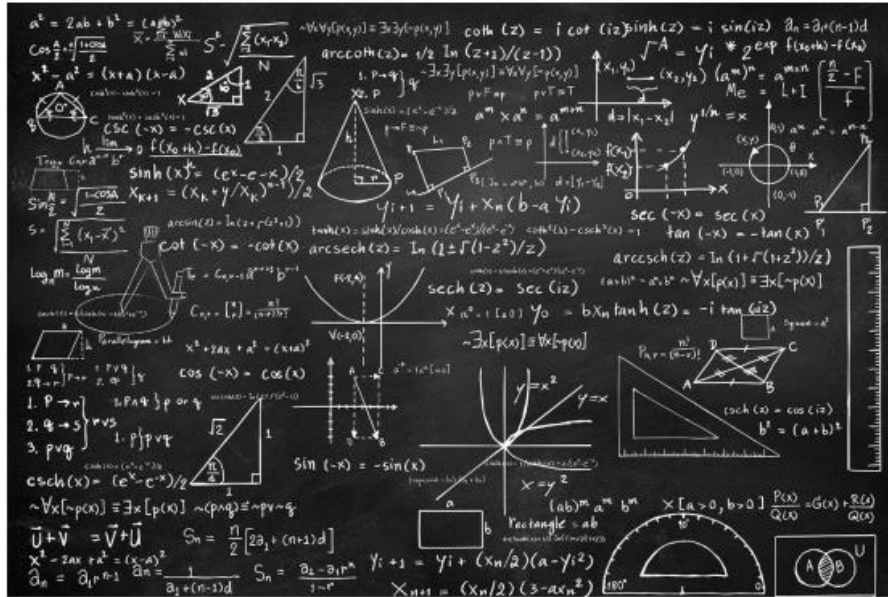


Wiskunde Opsommings

Gr 12



Vrywaring:

Die skrywer van die opsommings behou alle regte daarop en die opsommings mag slegs deur 123Tutors verkoop en versprei word.

Indien daar enige foute of onduidelikhede is, kontak asseblief 123Tutors sodat hul die boodskap kan deurgee aan die skrywer en dat dit reggestel kan word.

Enige bladsy nommers of voorbeelde verwys na die Platinum Wiskunde Leerderboek vir Gr12.

Die opsommings is nie daar om handboeke of klaswerk te vervang nie. Terugverwysing na handboeke is belangrik.

****PREVIEW****



123tutors.co.za/notes

$$\textcircled{2} \quad x^2 + 10x + k$$

Volkome vierkant $(x+p)^2$

$$= (x+5)^2$$

$$= x^2 + 10x + 25$$

$$x^2 + 10x + \left(\frac{10}{2}\right)^2 - \left(\frac{10}{2}\right)^2 + k = 0$$

$$(x+5)^2 - 25 + k = 0$$

$$-25 + k = 0$$

$$k = 25$$

$$\textcircled{3} \quad x^2 + bx + c = 0$$

Wortels = -5 of 7

$$x = -5 \text{ of } x = 7$$

$$(x+5) = 0 \text{ of } (x-7) = 0$$

$$(x+5)(x-7) = 0$$

$$x^2 - 2x - 35 = 0$$

$$b = -2 \quad c = -35$$

$$x^2 - 2x - 35 = 0$$

$$\textcircled{4} \quad x = -\frac{1}{2}$$

$$6x^2 + px - 4 = 0$$

$$6\left(-\frac{1}{2}\right)^2 + p\left(-\frac{1}{2}\right) - 4 = 0$$

$$\frac{3}{2} - \frac{1}{2}p - 4 = 0$$

$$-\frac{1}{2}p = 4 - \frac{3}{2}$$

$$-\frac{1}{2}p = \frac{5}{2}$$

$$p = -5$$

$$6x^2 - 5x - 4 = 0$$

$$(3x-4)(2x+1) = 0$$

$$x = \frac{4}{3} \text{ of } x = -\frac{1}{2}$$

PREVIEW



WORTELS

$$\begin{aligned} \textcircled{1} \quad x - \sqrt{-9x - 17} &= -3 \\ -\sqrt{-9x - 17} &= -3 - x \\ (\sqrt{-9x - 17})^2 &= (3 + x)^2 \\ -9x - 17 &= 9 + 6x + x^2 \\ 0 &= x^2 + 15x + 26 \\ 0 &= (x + 2)(x + 13) \\ x &= -2 \quad \text{of} \quad x = -13 \\ &\quad \text{N.V.T} \end{aligned}$$

* Kry wortel alleen
 $\div (-1)$
 Kwadreer

* TOETS: wortel
 mag nooit '-'
 wees nie.

$$\begin{aligned} \textcircled{2} \quad 3\sqrt{x+4} &= x \\ (\sqrt{x+4})^2 &= \left(\frac{x}{3}\right)^2 \\ x+4 &= \frac{x^2}{9} \\ 9(x+4) &= x^2 \\ 9x + 36 - x^2 &= 0 \\ x^2 - 9x - 36 &= 0 \\ 0 &= (x - 12)(x + 3) \\ x &= 12 \quad \text{of} \quad x = -3 \\ &\quad \text{N.V.T} \end{aligned}$$

$$\text{TOETS: } \textcircled{12} \quad 3\sqrt{12+4} = 12$$

$$12 = 12$$

$$\textcircled{-3} \quad 3\sqrt{4-3} = -3$$

$$3 \neq -3 \quad \text{N.V.T}$$

Moet dieselfde wees

$$\begin{aligned} \textcircled{3} \quad 6 - \sqrt{x+2} &= x+2 \\ -\sqrt{x+2} &= x-4 \\ (\sqrt{x+2})^2 &= (-x+4)^2 \\ x+2 &= x^2 - 8x + 16 \\ x^2 - 9x + 14 &= 0 \\ x &= 7 \quad \text{of} \quad x = 2 \\ &\quad \text{N.V.T} \end{aligned}$$

(Deel deur '-')

PREVIEW



b.3. $2; 7; 2; 10; 2; 13; 2; 16$

a. $7; 10; 13; 16...$

Ewe terme:
 $T_n = 7 + 3(n-1)$
 $= 3n + 4$

$2; 2; 2; 2...$

Onewe terme:
 $T_n = 2$

b. Bepaal term: -100

2, want onewe terme bly 2.

③ $3; 8; 15; 24; 35; 48$

$$\begin{array}{ccccccc} & \underbrace{\quad} & \underbrace{\quad} & \underbrace{\quad} & \underbrace{\quad} & \underbrace{\quad} & \\ 5 & 7 & 9 & 11 & 13 & & \\ & \underbrace{\quad} & \underbrace{\quad} & \underbrace{\quad} & \underbrace{\quad} & & \\ & 2 & 2 & 2 & 2 & & \end{array}$$

$$T_n = an^2 + bn + c$$

$$T_n = n^2 + 2n$$

$$a = \frac{2}{2} = 1$$

$$b = 5 - 3(1)$$

$$= 2$$

$$c = 3 - 1 - 2$$

$$= 0$$

Watter term = 624

$$624 = n^2 + 2n$$

$$0 = n^2 + 2n - 624$$

$$0 = (n - 24)(n + 26)$$

$$n = -26 \quad \underline{n = 24} \rightarrow$$

N.V.T

PREVIEW



Nominale koers $\leftrightarrow$ Effektiewe koers

$$\rightarrow 1 + i_{\text{eff}} = \left(1 + \frac{i^{(m)}}{m} \right)^m$$

nominale rentekoers

m

$\rightarrow$ aantal kere rente in 1 jaar saamgestel word.

Tydlyn

Mag nie afrond tot laaste antw. nie.

$\rightarrow$ In grafiek is enkelvoudig-reguitlyn
saamgestel-eksponensiëel

Delgingsfondse $\rightarrow$ verkoop oud - koop nuut: spaarplan vir geld benodig.

Annuïteite $\rightarrow$ Reeks gelyke betalings wat gereed gemaak word.

Leen

$\rightarrow$ spaar

$$FV = \frac{a \left[(1+i)^n - 1 \right]}{i}$$

Future value

vorm gereelde betaling
vorm meetkundig reeks

PREVIEW



$f(x) < 0 \rightarrow$ waar helling
 $\ominus$ is.

Koëffisiënte v. funksies

$$f(x) = ax^3 + bx^2 + cx + d$$

$$f(1) = f'(1) = 0$$

$$f(0) = 4$$

$$f'(0) = -7$$

$$f'(x) = 3ax^2 + 2bx + c$$

$$\underline{d = 4} \rightarrow$$

$$\underline{c = -7} \rightarrow$$

$$f(1) = d(1)^3 + b(1)^2 + (-7) + 4 = 0$$

$$= a + b - 3 = 0$$

$$a = 3 - b$$

$$f'(1) = 3a(1)^2 + 2b(1) - 7$$

$$= 3a + 2b - 7 = 0$$

$$3a = -2b + 7$$

$$3(3 - b) = -2b + 7$$

$$9 - 3b = -2b + 7$$

$$-1b = -2$$

$$b = 2$$

$\rightarrow$

$$a = 3 - b$$

$$a = 3 - 2$$

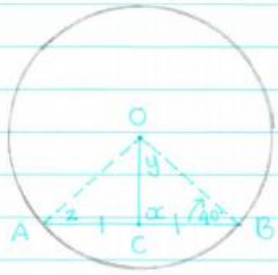
$$a = 1$$

$\rightarrow$

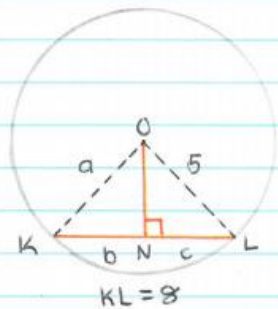
****PREVIEW****



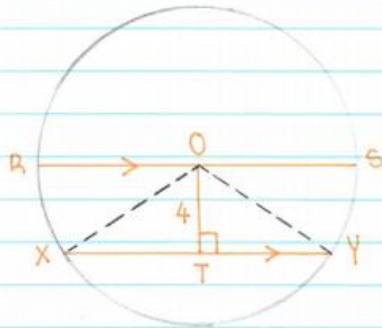
Ub. 3.

 $\hat{B} = 40^\circ \dots$ Gegee $x = 90^\circ \dots$ (midpt O, midpt koord) $y = 50^\circ$ $180^\circ - 90^\circ - 40^\circ = y \dots$ (binne $\angle$ v. Δ)
 $= 50^\circ$ $\hat{B} = z = 40^\circ$ $OA = OB = \text{radius} \dots$ (gelyke $\angle$ teenoor gelyke sye)

Ub. 4.

 $KL = 8 \dots$ Gegee $c = b = 4 \dots$ ewewydig (midpt O, midpt koord) $\therefore 8 : 2 = 4$ $a = 5 = \text{radius} \dots$ (gelyke $\angle$, teenoor gelyke sye.)

Ub. 5.

 $XY = 12 \text{ cm} \dots$ (gegee) $XT = TY = 6 \text{ cm} \dots$ (midpt O, $\perp$ koord) $OX = OY = RO = OS = \text{radius}$ $OX^2 = OT^2 + TX^2 \dots$ Pyth $OX^2 = 4^2 + 6^2$ $OX^2 = 52$ $OX = 2\sqrt{13} \text{ cm}$ $\therefore RO = OS = 2\sqrt{13}$ $\therefore 2\sqrt{13} + 2\sqrt{13} = RS$ $\therefore RS = 4\sqrt{13} \text{ cm}$

PREVIEW

Nominale koers ↔ Effektiewe koers

$$\rightarrow 1 + i_{\text{eff}} = \left(1 + \frac{i^{(m)}}{m} \right)^m$$

$i^{(m)}$ ← nominale rentekoers
 m ← aantal kere rente in 1 jaar saamgestel word.

Tydlyn

Mag nie afrond tot laaste antw. nie.

→ In grafiek is enkelvoudig-reguitlyn
saamgestel - eksponensieel

Delgingsfondse → verkoop oud - koop nuut: spaarplan vir geld benodig.

Annuïteite → Reeks gelyke betalings wat gereed gemaak word.

↓
Leen

→ spaar

$$FV = \frac{P \left[(1+i)^n - 1 \right]}{i}$$

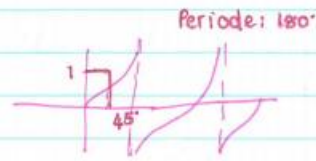
P ← vorm gereelde betaling
 FV ← Future value
 $\left[(1+i)^n - 1 \right]$ ← vorm meetkundige reeks

PREVIEW



→ Teken elke 30°

$$y = \tan x$$



Ankerpunt (elke 45°)

$$y = -3 \tan(2x - 45^\circ) + 1$$

refleksie links of regs transleer op en af transleer

Periode $\frac{180^\circ}{2} = 90^\circ$

Normaal

$(45^\circ; 1) \rightarrow (45^\circ; -3)$ $45^\circ \rightarrow$ punt op y-as

Asimptote: $x = 90^\circ + k \cdot 180^\circ$

↳ hoeveel keer herhaal

KEIR

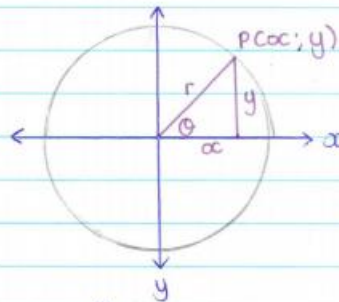
★ bl. 116 - 119 → standaard

Identiteite

$$\sin \theta = \frac{y}{r}$$

$$\cos \theta = \frac{x}{r}$$

$$\tan \theta = \frac{y}{x}$$



$$x^2 + y^2 = r^2 \quad \text{Pyth}$$

① $\tan \theta = \frac{\sin \theta}{\cos \theta}$ Bewys: Rk = $\frac{\sin \theta}{\cos \theta}$

$$= \frac{y/r}{x/r}$$

$$= \frac{y}{r} \times \frac{r}{x}$$

$$= \frac{y}{x}$$

$$= \tan \theta$$

Alternatiewe:

$$\# \tan \theta \times \cos \theta = \sin \theta$$

$$\# \cos \theta = \frac{\sin \theta}{\tan \theta}$$

$$\# \frac{1}{\tan} = \frac{\cos \theta}{\sin \theta}$$

PREVIEW

KO-FUNKSIES → maak saam 90°

$$\rightarrow \sin(90^\circ - \theta) = \cos \theta$$

$$\text{bewys: } \sin(90^\circ - \theta) = \frac{a}{r} = \cos \theta$$

$$\rightarrow \cos(90^\circ - \theta) = \sin \theta$$

$$\therefore \cos(90^\circ - \theta) = \frac{b}{r} = \sin \theta$$

$$\rightarrow \sin(90^\circ + \theta) = \cos \theta$$

$$\therefore \sin(90^\circ + \theta) = \frac{a}{r} = \cos \theta$$

$$\rightarrow \cos(90^\circ + \theta) = -\sin \theta$$

$$\therefore \cos(90^\circ + \theta) = -\frac{b}{r} = -\sin \theta$$

REDUKSIE FORMULE'S

$$\left(\begin{array}{l} -\sin(180^\circ - \theta) = \sin \theta \\ -\cos(180^\circ - \theta) = -\cos \theta \\ -\tan(180^\circ - \theta) = -\tan \theta \end{array} \right\}$$

$$180^\circ - \theta$$

→ 2de
Kwadrant
→ sin (+)

$$\left(\begin{array}{l} -\sin(180^\circ + \theta) = -\sin \theta \\ -\cos(180^\circ + \theta) = -\cos \theta \\ -\tan(180^\circ + \theta) = \tan \theta \end{array} \right\}$$

$$180^\circ + \theta$$

→ 3de
Kwadrant
→ tan (+)

$$\left(\begin{array}{l} -\sin(360^\circ - \theta) = -\sin \theta \\ -\cos(360^\circ - \theta) = \cos \theta \\ -\tan(360^\circ - \theta) = -\tan \theta \end{array} \right\}$$

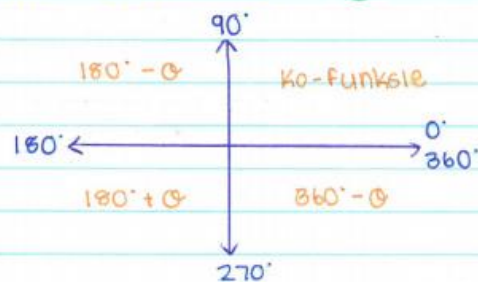
$$360^\circ - \theta$$

→ 4e Kwadrant
→ cos (+)

$$\left(\begin{array}{l} -\sin(360^\circ + \theta) = \sin \theta \\ -\cos(360^\circ + \theta) = \cos \theta \\ -\tan(360^\circ + \theta) = \tan \theta \end{array} \right\}$$

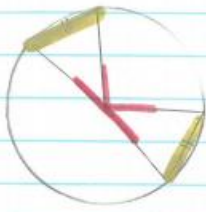
$$360^\circ + \theta$$

→ 1ste
Kwadrant
→ Almal (+)



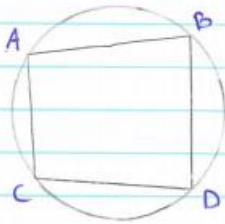
PREVIEW





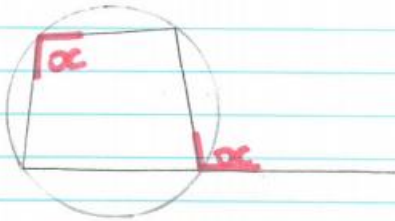
$\angle$ by midpt v. O, onderspan deur = e koorde is =.

Stelling 5:



Teenoorstaande $\angle$ v. KUH is ooplimentêr.

5b:



Buite $\angle$ v. KUH is = aan teenoorstaande binne $\angle$.

Raaklyn:



Raaklyn $\perp$ radius

PREVIEW

3D-TRIG



Sin-reël:

$$\text{Gy: } \frac{a}{\sin \hat{A}} = \frac{b}{\sin \hat{B}} = \frac{c}{\sin \hat{C}}$$

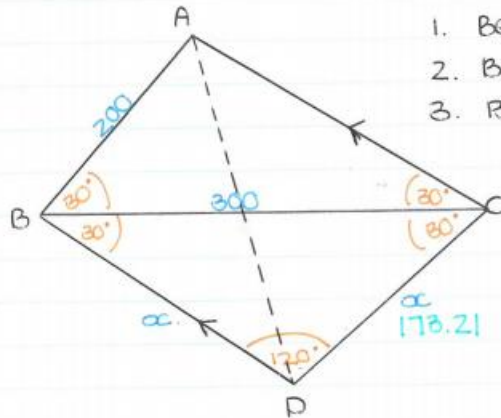
$$\text{Hoek: } \frac{\sin \hat{A}}{a} = \frac{\sin \hat{B}}{b} = \frac{\sin \hat{C}}{c}$$

Cos-reël:

$$b^2 = a^2 + c^2 - 2ac \cos \hat{B}$$

Oppv-reël:

$$\frac{1}{2} \cdot a \cdot b \cdot \sin \hat{C}$$

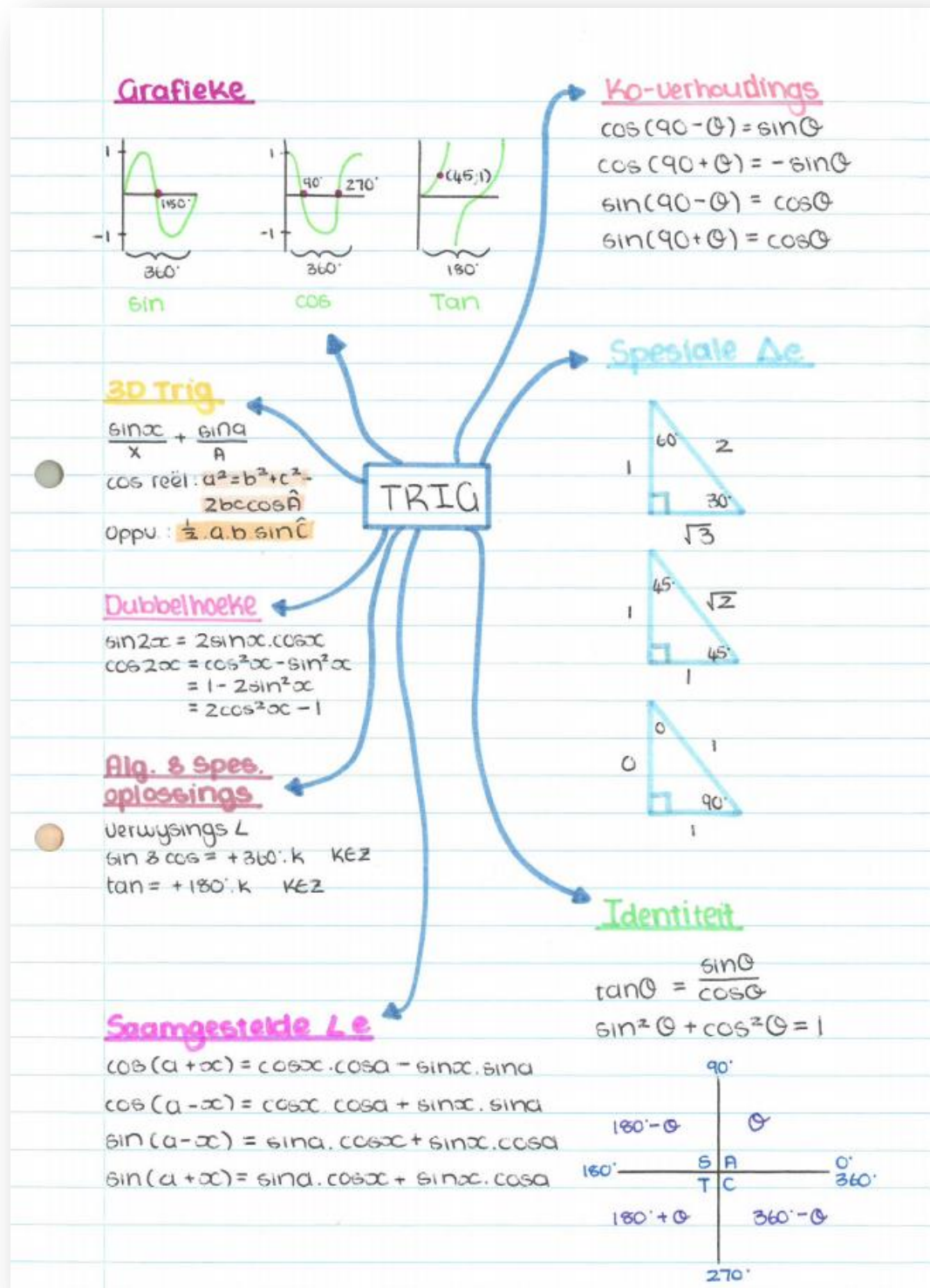
Ub 1

1. Bereken BC
2. Bereken α
3. Bereken $\hat{A}$
4. Oppv.

$$\begin{aligned} 1. \quad BC^2 &= DB^2 + DC^2 - 2(DB)(DC)\cos 120^\circ \\ BC^2 &= \alpha^2 + \alpha^2 - 2(\alpha)(\alpha)\cos 120^\circ \\ BC^2 &= 2\alpha^2 - 2\alpha^2\cos 120^\circ \\ BC^2 &= 2\alpha^2 - 2\alpha^2\left(-\frac{1}{2}\right) \\ BC^2 &= 2\alpha^2 + \alpha^2 \\ BC &= \sqrt{3}\alpha \end{aligned}$$

PREVIEW





PREVIEW